AIAA JOURNAL
Vol. 37, No. 9, September 1999

Numerical Investigation of the Noise Radiated from Hot Subsonic Turbulent Jets

Véronique Fortuné* and Yves Gervais†
Université de Poitiers, 86022 Poitiers CEDEX, France

Experimental studies have highlighted the existence of additional sources of sound due to temperature fluctuations in heated jets, in comparison with unheated jets. Whereas cold jets have been the subject of many numerical investigations, little research has been devoted to the topic of hot jets. Thus, a specific model to investigate the acoustic radiation from these jets based on numerical predictions using a k–ε turbulence closure is attempted. First, a model for the additional source term is suggested, which is a function of mean axial velocity, mean temperature, and turbulent kinetic energy. Next a computation model of acoustic intensity spectrum is developed in which three different contributions appear: The first represents the contribution of the velocity fluctuations in the sound emission, the second that of the temperature fluctuations, and the third a mixed term issuing from the two preceding contributions. Then the model is applied to compute the acoustic radiation of hot jets. Results provided quite a full description of the acoustic features of a hot jet: spectrum shape, radiated acoustic intensity levels, and influence of jet temperature. Finally, the model seemed capable of predicting the trends in noise radiation of turbulent hot jets correctly.

Nomenclature

A = instantaneous acoustic source term related to velocity fluctuations
$\hat{A}$ = root mean square amplitude of A
C = convection factor
C_p = specific heat at constant pressure
C_{pp} = normalized pressure autocorrelation function
c = sound speed
D = exit nozzle diameter
E = spatiotemporal intercorrelation factor
f = frequency
I = acoustic intensity
k = turbulent kinetic energy
$\boldsymbol{k}$ = acoustic wave vector
L = turbulence length scale
M_c = convection Mach number
$M_{i,j}$ = local source point
M_0 = observation point
p, p' = pressure, acoustic pressure fluctuation
S = acoustic source term related to temperature fluctuations
$\hat{S}$ = root mean square amplitude of S
Sr = Strouhal number
s = entropy
T = temperature
T_j = jet exit temperature
t = time
U_c = convection velocity
U_j = jet exit velocity
u_i = fluid velocity components
V = acoustic source volume
$\boldsymbol{x}$ = vector related to the observer position
$\boldsymbol{y}, \boldsymbol{y}', \boldsymbol{y}''$ = vectors related to the source points
Γ = spatiofrequential coherence function
δ_{ij} = Kronecker delta function
ε = dissipation rate of the turbulent kinetic energy
$\boldsymbol{\eta}$ = separating vector of source points
$\boldsymbol{\xi}$ = separating vector of source points in the moving frame
ρ, ρ' = density, acoustic density fluctuation
τ_{ij} = viscous stress tensor
ω = angular frequency
ω_t = turbulence characteristic angular frequency

Subscripts

r = in the radial direction
x = in the axial direction
0 = in the medium at rest

I. Introduction

THIS work is situated in the general context of research on noise emission from aerospace launchers. It is well known that the payload is likely to be damaged because of noise emissions from jet engines during takeoff. The jet engines of launchers such as Ariane are strongly supersonic and heated (typically, $U_j/c_0 \approx 7$ and $T_j \approx 1800$ K). Within the past 10 years, the Centre National d'Etudes Aérospatiales has been at the origin of many overall studies relative to subsonic and supersonic turbulent jets, with a view to improve the understanding of the physical mechanisms of aerodynamic noise generation. Thus, experimental studies on launcher models such as the Banc MARTEL[1] are carried out, and moreover, it is of great interest and also less expensive to work out numerical models capable of predicting acoustic radiation of freejets, even if their physical characteristics seem remote from those of real launchers.

Acoustic radiation of ambient temperature turbulent freejets has been widely studied from experimental (for example, by Lush[2] and Tanna[3]) and theoretical[4,5] viewpoints. More recently, Béchara et al.[6] and Bailly et al.[7] developed numerical models to predict the acoustic emission of cold turbulent jets using information on the mean aerodynamic field obtained from Reynolds averaged Navier–Stokes equation solver associated with a k–ε closure. Unfortunately, there is little research related to the noise from jets whose mean static temperature differs from the ambient. However, it is still possible to point out general characteristics of the noise generated by hot free turbulent jets thanks to some experimental studies. At first, as shown by Fisher et al.,[8] who have made a general review of the literature on this subject, for a constant mean velocity of the jet, a variation in the mean temperature of the jet increases the

Presented as Paper 98-2356 at the AIAA 4th Aeroacoustics Conference, Toulouse, France, 2–4 June 1998; received 10 August 1998; revision received 1 February 1999; accepted for publication 5 February 1999.

*Ph.D. Student, Laboratoire d'Etudes Aérodynamiques, UMR Centre National de la Recherche Scientifique 6609, Bâtiment H, 40 Avenue du Recteur Pineau.

†Professor, Laboratoire d'Etudes Aérodynamiques, UMR Centre National de la Recherche Scientifique 6609, Bâtiment H, 40 Avenue du Recteur Pineau.

acoustic level if the velocity of the jet is such that its Mach number is lower than 0.7 and decreases the level if the Mach number is larger than 0.7. Furthermore, spectrum measurements made by Tanna[3] and Tanna et al.[9] on cold and hot subsonic and supersonic jets show that the acoustic intensity radiated from the hot jet is higher than the one radiated from the cold jet in the low-frequencies domain, evidently involving a shift of the hot jet spectrum central frequency in comparison to the cold one. These results clearly indicate that the acoustic emission of hot jets is fundamentally different from that of cold jets. Indeed, there appears an additional physical mechanism of noise emission in hot turbulent jets related to the difference in mean temperature between the jet and the ambient fluid.

Hence, there is interest in developing a numerical model that is able to correctly give the acoustic emission from hot jets. First we mention the approach due to Lilley,[10] who suggested a formulation derived from that of Lighthill, which included the total energy relation in the wave equation. This model was applied to the calculation of the acoustic emission of cold and hot jets and allowed direct evaluation of the radiated acoustic power using results from direct numerical simulations of isotropic homogeneous turbulence.[11]

Here we try to develop a predictive model of the noise radiated from a hot subsonic air circular freejet using statistical quantities obtained from a fluid mechanic turbulent k–ε computation code. Our approach is still based on the Lighthill aeroacoustic analogy, which has the advantage of having a simple far-field solution in free space as soon as the source term is known. The first step consists of identifying acoustic source terms, i.e., those due to the turbulence of the flow and those related to the difference in mean temperature between the jet and the ambient. Afterward, the evaluation of the acoustic emission is made from the statistical approach generally adopted for the calculation of the acoustic radiation of free turbulent cold flows. Then the model is applied to calculate the acoustic spectrum of different temperature and velocity jets, where root mean square amplitudes of source terms are estimated from mean and statistical characteristics of the flow given by the k–ε model. Finally, a general discussion of our numerical results is presented.

II. Hot Jet Noise Modeling

As stated in the Introduction, the model is based on Lighthill's acoustic analogy. By combining the mass and momentum conservation equations, one obtains the wave equation for the density ρ. As the wave operator refers to the ambient, i.e., an homogeneous medium at rest, we can directly write the wave equation for the acoustic pressure:

$$\frac{1}{c_0^2}\frac{\partial^2 p}{\partial t^2} - \Delta p = \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j} \tag{1}$$

where T_{ij} is the tensor of Lighthill,

$$T_{ij} = \rho u_i u_j + \left(p - c_0^2\rho\right)\delta_{ij} - \tau_{ij}$$

The solution of the Lighthill equation in free space and far field is obtained from the associated Green's function, which yields

$$p'(\boldsymbol{x}, t) = \int_V \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j}\left(\boldsymbol{y}, t - \frac{|\boldsymbol{x}-\boldsymbol{y}|}{c_0}\right)\frac{\mathrm{d}\boldsymbol{y}}{4\pi|\boldsymbol{x}-\boldsymbol{y}|} \tag{2}$$

where p' represents the acoustic pressure fluctuations at the observation point M_0 due to the turbulent volume V, respectively located at $\boldsymbol{x}$ and $\boldsymbol{y}$. Using the far-field assumption again, Goldstein[12] shows that this may be expressed as

$$p'(\boldsymbol{x}, t) = \frac{x_i x_j}{c_0^2|\boldsymbol{x}|^2}\frac{\partial^2}{\partial t^2}\int_V [\rho u_i u_j - \tau_{ij}]\frac{\mathrm{d}\boldsymbol{y}}{4\pi|\boldsymbol{x}|} + \frac{\partial}{\partial t}\int_V\left[\frac{\partial}{\partial t}\left(\frac{1}{c_0^2}p - \rho\right)\right]\frac{\mathrm{d}\boldsymbol{y}}{4\pi|\boldsymbol{x}|} \tag{3}$$

where quantities in brackets are taken at time $t - |\boldsymbol{x}-\boldsymbol{y}|/c_0$.

This last relation clearly shows two distinct contributions to the noise emission of a hot jet. The first one is related to the instantaneous fluctuations of velocity and viscous stress. For a subsonic, high-Reynolds-number airflow expanding in an homogeneous medium at rest, the acoustic contribution of viscous stress is known to be negligible in comparison to that of velocity.[4] The second important source results from nonisentropic turbulent heat transfer. This term had to be defined explicitly.

For simplicity, we introduce the following notations for the acoustic source terms:

$$A(\boldsymbol{y}, t) = \rho u_i u_j(\boldsymbol{y}, t), \qquad S(\boldsymbol{y}, t) = \frac{\partial}{\partial t}\left(\frac{1}{c_0^2}p - \rho\right)(\boldsymbol{y}, t) \tag{4}$$

The expression of the acoustic pressure radiated from the source volume V at the observation point M_0 becomes

$$p'(\boldsymbol{x}, t) = \frac{1}{4\pi|\boldsymbol{x}|}\left\{\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\int_V A\left(\boldsymbol{y}, t - \frac{|\boldsymbol{x}-\boldsymbol{y}|}{c_0}\right)\mathrm{d}\boldsymbol{y} + \frac{\partial}{\partial t}\int_V S\left(\boldsymbol{y}, t - \frac{|\boldsymbol{x}-\boldsymbol{y}|}{c_0}\right)\mathrm{d}\boldsymbol{y}\right\} \tag{5}$$

A. Acoustic Source Terms Modeling

In this section, we give a statement of the root mean square amplitude of the acoustic sources from the mean flow quantities. Let us call $\hat{A}(\boldsymbol{y})$ and $\hat{S}(\boldsymbol{y})$ the rms amplitudes of the instantaneous aerodynamic and entropic sources $A(\boldsymbol{y}, t)$ and $S(\boldsymbol{y}, t)$, respectively. We just have shown the two origins of the noise radiated by a hot jet. As this type of noise source was of no interest to us, we adopt a very simple expression that only takes into account the noise due to the turbulence–turbulence interactions. Our main focus is on the entropic noise source term definition.

1. *Aerodynamic Source Term*

According to the Reynolds decomposition, the rms value of the aerodynamic source term is

$$\overline{\rho u_i u_j} = \bar{\rho}\left(\overline{U_i U_j} + \overline{U_i u'_j} + \overline{U_j u'_i} + \overline{u'_i u'_j}\right) \tag{6}$$

with $\bar{\rho}$ the local mean density.

The first term does not contribute to noise generation, the last term represents noise generated by turbulence–turbulence interactions, often called self-noise,[12] whereas the two remaining terms represent noise generated by turbulence–mean flow interactions, usually referred to as shear noise.[12] As already stated, for simplicity we chose to retain the contribution of the turbulence–turbulence interactions only and to keep the local mean density for accounting for density variations within the jet. Finally, we approximate the velocity correlation from the kinetic turbulent energy k (provided by the k–ε turbulent model), and the following expression is retained to estimate the aerodynamic acoustic contribution:

$$\hat{A}(\boldsymbol{y}) = \overline{\rho u_i u_j}(\boldsymbol{y}) \approx \bar{\rho}\left(\tfrac{2}{3}k\right)(\boldsymbol{y}) \tag{7}$$

2. *Entropic Source Term*

We try here to express explicitly the relation between this last kind of source and entropy fluctuations. This is made from a development similar to that of Morfey,[13] who suggested to split the term $\partial(p/c_0^2 - \rho)/\partial t$ to define all of the effects it contains separately. A general discussion about the influence of all of these effects on the acoustic emission yields a good approximation of the entropic source term.

Following Morfey,[13] the first step consists in giving an expression of the entropic source in terms of material derivative of the pressure and density:

$$\frac{\partial}{\partial t}\left(\frac{1}{c_0^2}p - \rho\right) = \frac{1}{c_0^2}\frac{\mathrm{d}p}{\mathrm{d}t} - \frac{\mathrm{d}\rho}{\mathrm{d}t} - \frac{\partial}{\partial x_i}\left[u_i\left(\frac{1}{c_0^2}(p - p_0) - (\rho - \rho_0)\right)\right] + \left(\frac{1}{c_0^2}(p - p_0) - (\rho - \rho_0)\right)\frac{\partial u_i}{\partial x_i} \tag{8}$$

and combining this with a state relation [such as $\rho = \rho(p, s)$ and assuming a perfect gas] and the mass conservation law. This yields

$$\frac{\partial}{\partial t}\left(\frac{1}{c_0^2}p - \rho\right) = \left(\frac{1}{c_0^2} - \frac{1}{c^2}\right)\frac{\mathrm{d}p}{\mathrm{d}t} + \frac{\rho_0}{C_p}\frac{\mathrm{d}s}{\mathrm{d}t} + \frac{1}{c^2}\frac{(\rho - \rho_0)}{\rho}\frac{\mathrm{d}p}{\mathrm{d}t} + \frac{1}{c_0^2}(p - p_0)\left(-\frac{1}{\rho c^2}\frac{\mathrm{d}p}{\mathrm{d}t} + \frac{1}{C_p}\frac{\mathrm{d}s}{\mathrm{d}t}\right) - \frac{\partial}{\partial x_i}\left[u_i\left(\frac{1}{c_0^2}(p - p_0) - (\rho - \rho_0)\right)\right] \tag{9}$$

A dimensional analysis is then conducted to extract the dominant phenomena contained in this expression. As suggested by Morfey,[13] $1/c_0$ and $1/c$ can be formally regarded as small parameters. Thus, one can consider as negligible the contribution of the terms of $\mathcal{O}(1/c_0^2)$ or $\mathcal{O}(1/c^2)$ in the right-hand side of Eq. (9) on the far-field acoustic pressure. Furthermore, according to the far-field approximation to estimate the acoustic contribution of the last term of the right-hand side of Eq. (9), it appears that only the term $\rho_0(\mathrm{d}s/\mathrm{d}t)/C_p$ has a significant contribution. This allows the following approximation of the entropic acoustic source:

$$\frac{\partial}{\partial t}\left(\frac{1}{c_0^2}p - \rho\right) \approx \frac{\rho_0}{C_p}\frac{\mathrm{d}s}{\mathrm{d}t} \tag{10}$$

B. First-Order Closure for Entropy Fluctuations Evaluation

We now give an expression of the acoustic term related to the entropy fluctuations only as a mean statistical quantities function of the flow. It is possible to obtain neither temperature fluctuations nor an equivalent quantity from a classical turbulent code, so that the numerical computation of these fluctuations has to be made according to a specific closure model. Following Berman,[14] modeling of the entropic acoustic source term is obtained by considering the first and second thermodynamic principles for a perfect gas and assuming constant pressure evolution, an assumption which remains valid as long as we consider a jet expanding in free space. Absolute entropy and temperature could simply be related by

$$\frac{1}{C_p}\frac{\mathrm{d}s}{\mathrm{d}t} = \frac{1}{T}\frac{\mathrm{d}T}{\mathrm{d}t} \tag{11}$$

From a statistical point of view and according to Reynolds decomposition, it is still possible to express the time average of the material derivative of the temperature:

$$\overline{\frac{\mathrm{d}T}{\mathrm{d}t}} = \bar{U}_i\frac{\partial\bar{T}}{\partial x_i} + \overline{u_i'\frac{\partial T'}{\partial x_i}} \tag{12}$$

As the noise from entropy fluctuations is to be related to turbulent heat flux, the time average of entropic acoustic sources is given by

$$\frac{1}{C_p}\overline{\frac{\mathrm{d}s}{\mathrm{d}t}} \approx \frac{1}{\bar{T}}\overline{u_i'\frac{\partial T'}{\partial x_i}} \tag{13}$$

Assuming a quasi-unidimensional free flow and according to the thin-layer approximations, we can now develop the entropic acoustic source term as a function of the velocity temperature velocity correlation:

$$\frac{1}{C_p}\overline{\frac{\mathrm{d}s}{\mathrm{d}t}} \approx \frac{1}{\bar{T}}\overline{\frac{\partial u_i'T'}{\partial x_i}} \approx \frac{1}{\bar{T}}\overline{\frac{\partial u_r'T'}{\partial r}} \tag{14}$$

where r is the direction perpendicular to the jet axis.

It is now necessary to introduce a closure model to compute the velocity–temperature correlation from quantities given by a turbulent k–ε code. Here we retain a first-order closure model, as suggested by Berman[14] and Fulachier and Dumas[15]:

$$\left.\begin{aligned}\overline{u_x'u_r'} &= \alpha\frac{\partial\bar{U}_x}{\partial r}\\ \overline{u_r'T'} &= \alpha\frac{\partial\bar{T}}{\partial r}\end{aligned}\right\} \Rightarrow \overline{u_r'T'} = \overline{u_x'u_r'}\left(\frac{\partial\bar{T}}{\partial r}\bigg/\frac{\partial\bar{U}_x}{\partial r}\right) \tag{15}$$

where $\bar{U}_x$ is the mean flow velocity in the jet direction, which implicitly considers a strong correlation between temperature (or entropy) and velocity fluctuations. The acoustic source term due to entropy fluctuations is then expressed as

$$\frac{\rho_0}{C_p}\overline{\frac{\mathrm{d}s}{\mathrm{d}t}} \approx \rho_0\frac{1}{\bar{T}}\frac{\partial}{\partial r}\left[\overline{u_x'u_r'}\left(\frac{\partial\bar{T}}{\partial r}\bigg/\frac{\partial\bar{U}_x}{\partial r}\right)\right] \tag{16}$$

Though the entropic acoustic term as just formulated appears to be only a function of statistical quantities of the flow (mean velocity and temperature and velocity correlation), such a modeling is not sufficient because one recalls that a k–ε model, based on an isotropic description of the turbulence, only gives the kinetic energy k, whereas the correlation $\overline{u_x'u_r'}$ is needed. In practice, this correlation could still be related to the kinetic energy according to experimental investigations due to Abramovich,[16] who showed that it was closely approximated by $k/3$.

Finally, we propose the following model of the entropic acoustic source term, which is entirely composed of mean flow properties, mean temperature and axial velocity, and turbulent kinetic energy:

$$\hat{S}(\boldsymbol{y}) = \frac{\rho_0}{C_p}\overline{\frac{\mathrm{d}s}{\mathrm{d}t}}(\boldsymbol{y}) \approx \rho_0\frac{1}{\bar{T}}\frac{\partial}{\partial r}\left[\frac{1}{3}k\left(\frac{\partial\bar{T}}{\partial r}\bigg/\frac{\partial\bar{U}_x}{\partial r}\right)\right](\boldsymbol{y}) \tag{17}$$

C. Hot Jet Noise Radiation Estimation

The statistical approach adopted actually implies that the computation of an instantaneous acoustic quantity is no longer possible. Only quadratic values such as rms acoustic pressure or acoustic intensity could still be worked out. We then develop a computation model of an acoustic quantity of order two, the acoustic intensity, using rms values of source terms. Our approach is based on that generally adopted for determining the noise radiated from cold jets and consists in evaluating the acoustic pressure autocorrelation function in terms of the rms values of source terms, taking into account their interactions, and giving the acoustic intensity spectrum from a simple Fourier transform.[12]

1. *Acoustic Pressure Autocorrelation Function*

The normalized pressure autocorrelation function is defined as[12]

$$C_{pp}(\boldsymbol{x}, \tau) = \frac{1}{T}\int_t^{t+T}\frac{p'(\boldsymbol{x}, t)p'(\boldsymbol{x}, t-\tau)}{\rho_0 c_0}\,\mathrm{d}t \tag{18}$$

Upon assuming a stationary turbulence and using far-field approximation again, the autocorrelation function yields

$$C_{pp}(\boldsymbol{x}, \tau) = \frac{1}{16\pi^2|\boldsymbol{x}|^2\rho_0 c_0} \times \left\{\frac{1}{c_0^4}\frac{\partial^4}{\partial\tau^4}\iint\limits_{VV}\frac{1}{T}\int_t^{t+T}A(\boldsymbol{y}', t)A\left[\boldsymbol{y}'', t - \left(\tau - \frac{\boldsymbol{x}\cdot\boldsymbol{\eta}}{|\boldsymbol{x}|c_0}\right)\right]\mathrm{d}t\,\mathrm{d}\boldsymbol{y}'\,\mathrm{d}\boldsymbol{\eta} + \frac{2}{c_0^2}\frac{\partial^3}{\partial\tau^3}\iint\limits_{VV}\frac{1}{T}\int_t^{t+T}A(\boldsymbol{y}', t)S\left[\boldsymbol{y}'', t - \left(\tau - \frac{\boldsymbol{x}\cdot\boldsymbol{\eta}}{|\boldsymbol{x}|c_0}\right)\right]\mathrm{d}t\,\mathrm{d}\boldsymbol{y}'\,\mathrm{d}\boldsymbol{\eta} + \frac{\partial^2}{\partial\tau^2}\iint\limits_{VV}\frac{1}{T}\int_t^{t+T}S(\boldsymbol{y}', t)S\left[\boldsymbol{y}'', t - \left(\tau - \frac{\boldsymbol{x}\cdot\boldsymbol{\eta}}{|\boldsymbol{x}|c_0}\right)\right]\mathrm{d}t\,\mathrm{d}\boldsymbol{y}'\,\mathrm{d}\boldsymbol{\eta}\right\} \tag{19}$$

where $\eta = y'' - y'$ is the separating vector of points M_i and M_j located inside the source zone.

This expression highlights three different kinds of contribution: The first represents the contribution due to velocity fluctuations in the sound emission, the second the contribution due to temperature fluctuations, and the third a mixed term issuing from the two earlier contributions. It could still be expressed as a function the rms amplitudes of source terms, by introducing a spatiotemporal intercorrelation factor $E(\boldsymbol{y}', \eta, \tau - \boldsymbol{x} \cdot \eta/|\boldsymbol{x}|c_0)$, which represents the interactions between the contributions of each source volume. Note that the choice of the same intercorrelation factor for all contributions is justified by the modeling of the entropic source term [Eq. (17)], which involves a strong correlation between temperature and velocity fluctuations.

2. Acoustic Intensity Spectrum

The acoustic intensity spectrum is directly obtained by taking the Fourier transform of the autocorrelation function of the acoustic pressure[12]:

$$I(\boldsymbol{x}, \omega) = \int_{-\infty}^{+\infty} C_{pp}(\boldsymbol{x}, \tau) e^{-j\omega\tau}\, d\tau \tag{20}$$

It yields

$$I(\boldsymbol{x}, \omega) = \frac{1}{16\pi^2 |\boldsymbol{x}|^2 \rho_0 c_0} \times \left\{ \frac{(j\omega)^4}{c_0^4} \int_{-\infty}^{+\infty} \iint_{VV} \hat{A}(\boldsymbol{y}')\hat{A}(\boldsymbol{y}'')E(\boldsymbol{y}', \boldsymbol{\eta}, \tau) e^{-j\omega\tau} e^{j\boldsymbol{k}\cdot\boldsymbol{\eta}}\, d\boldsymbol{y}'\, d\boldsymbol{\eta}\, d\tau \right.$$
$$\left. + \frac{2(j\omega)^3}{c_0^2} \int_{-\infty}^{+\infty} \iint_{VV} \hat{A}(\boldsymbol{y}')\hat{S}(\boldsymbol{y}'')E(\boldsymbol{y}', \boldsymbol{\eta}, \tau) e^{-j\omega\tau} e^{j\boldsymbol{k}\cdot\boldsymbol{\eta}}\, d\boldsymbol{y}'\, d\boldsymbol{\eta}\, d\tau + (j\omega)^2 \int_{-\infty}^{+\infty} \iint_{VV} \hat{S}(\boldsymbol{y}')\hat{S}(\boldsymbol{y}'')E(\boldsymbol{y}', \boldsymbol{\eta}, \tau) e^{-j\omega\tau} e^{j\boldsymbol{k}\cdot\boldsymbol{\eta}}\, d\boldsymbol{y}'\, d\boldsymbol{\eta}\, d\tau \right\} \tag{21}$$

where $\boldsymbol{k} = \omega\boldsymbol{x}/c_0|\boldsymbol{x}|$ is the acoustic wave vector.

One of the main difficulties in computing acoustic intensity is that both retarded time and the characteristic turbulence time appear in this calculation. This problem is generally avoided by introducing a coordinate system that moves with the eddies.[12] It is possible in the case of subsonic flows, as shown by Goldstein,[12] to neglect the retarded time in comparison with the characteristic turbulence time, which here corresponds to the decay time of the turbulence and which is evidently maximal in the moving system. Therefore, the intensity spectrum becomes

$$I(\boldsymbol{x}, \omega) = \frac{1}{16\pi^2 |\boldsymbol{x}|^2 \rho_0 c_0}$$
$$\times \left\{ \frac{\omega^4}{c_0^4} \iint_{VV} \hat{A}(\boldsymbol{y}')\hat{A}(\boldsymbol{y}'')\Gamma(\boldsymbol{y}', \boldsymbol{\xi}, \omega)\, d\boldsymbol{y}'\, d\boldsymbol{\xi} \right.$$
$$- j\frac{2\omega^3}{c_0^2} \iint_{VV} \hat{A}(\boldsymbol{y}')\hat{S}(\boldsymbol{y}'')\Gamma(\boldsymbol{y}', \boldsymbol{\xi}, \omega)\, d\boldsymbol{y}'\, d\boldsymbol{\xi}$$
$$\left. - \omega^2 \iint_{VV} \hat{S}(\boldsymbol{y}')\hat{S}(\boldsymbol{y}'')\Gamma(\boldsymbol{y}', \boldsymbol{\xi}, \omega)\, d\boldsymbol{y}'\, d\boldsymbol{\xi} \right\} \tag{22}$$

where $\boldsymbol{\xi} = \boldsymbol{\eta} - \boldsymbol{i}U_c\tau$ is the separating vector of source points in the moving frame, with $\boldsymbol{i}$ a unit vector in the mean flow direction and U_c the convection velocity of eddies, and $\Gamma(\,)$ is a spatiofrequential coherence function, which describes the interactions between the source terms in the spectral domain. This function is strongly related to the spatial and frequential scales, which feature the turbulence of the flow, and the following section is devoted to its definition.

3. Spatiofrequential Coherence Function Modeling

The spatiofrequential coherence function is the Fourier transform of a spatiotemporal intercorrelation function of sound sources. This function features the spatial and frequential distributions of the heated jet. The scarcity of this type of data leads us to retain the usual Gaussian form suggested by Ribner[17] for unheated jets and to take into account the jet axisymmetry:

$$\Gamma(\boldsymbol{y}', \boldsymbol{\xi}, \omega) = \exp\left(\frac{-\pi\xi_x^2}{L_x^2}\right)\exp\left(\frac{-\pi\xi_r^2}{L_r^2}\right)\frac{\sqrt{\pi}}{\omega_t}\exp\left(\frac{-C^2\omega^2}{4\omega_t^2}\right) \tag{23}$$

with L_x the longitudinal integral scale, ξ_x the axial distance between the two source points, L_r the transversal integral scale, ξ_r the radial distance between the two source points, and $C = 1 - M_c \cos\theta$, where $M_c = U_c/c_0$ is the convection Mach number.

III. Numerical Computation of Acoustic Radiation of Subsonic Hot Jets

The aerodynamic field simulation is achieved using a k–ε turbulence closure, which provides us with the mean temperature and velocities, the turbulent kinetic energy, and its dissipation of a subsonic hot jet. First, we present the conditions of jet flow simulations, and numerical results are compared with measurements of Lau.[18] The study of the acoustic radiation begins by localization of sound sources. Then, the estimation of the spectral distribution is carried out by using the model worked out in the preceding section. Finally, we investigate the results provided by our acoustic model.

A. Aerodynamic Field Simulations

The aerodynamic field is obtained from a Reynolds-averaged Navier–Stokes equations solver associated with a k–ε closure.[19] We assume an axisymmetric jet, and consequently, the computational domain is two dimensional. Figure 1 shows the computational domain and the boundary conditions. The domain size is chosen for minimizing the boundary conditions effects on the flow. We use a 90×49 rectangular mesh with nonuniform spacing. Grid points are compressed in the mixing and transition regions. The inlet conditions U_j and T_j represent the air jet at the nozzle exit. On the outside boundary, the static pressure p_s is zero to allow the ambient fluid to be swept along by the moving fluid, and $T_0 = 273$ K.

Computation of a heated jet is carried out, and the numerical results are compared with the experimental data of Lau.[18] The nozzle diameter is $D = 50.8$ mm, the jet velocity $U_j = 136$ m/s, and its temperature $T_j = 786$ K. Figures 2a and 2b show the radial profiles of mean axial velocity and mean temperature, respectively.

As is shown in Figs. 2, the numerical predictions are close to experimental data. The turbulent kinetic energy is given in Fig. 3.

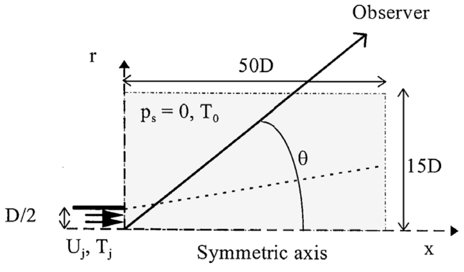

Fig. 1 Computational domain and boundary conditions of the jet flow simulation.

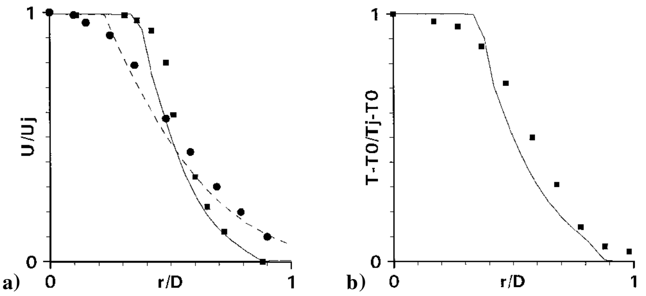

Fig. 2 Comparison between numerical profiles (——, $x/D = 2$ and - - -, $x/D = 4$) and Lau[18] measurements (■, $x/D = 2$ and ●, $x/D = 4$).

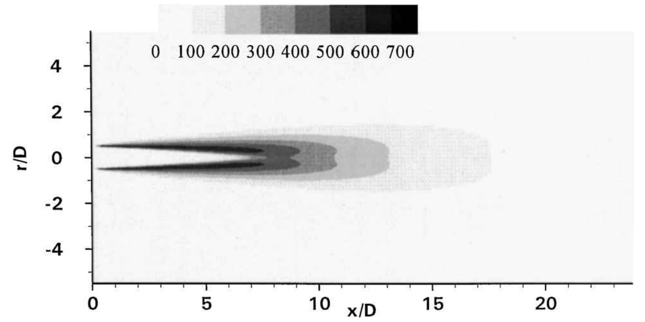

Fig. 3 Turbulent kinetic energy field of a heated jet: $D = 50.8$ mm, $U_j = 136$ m/s, and $T_j = 786$ K.

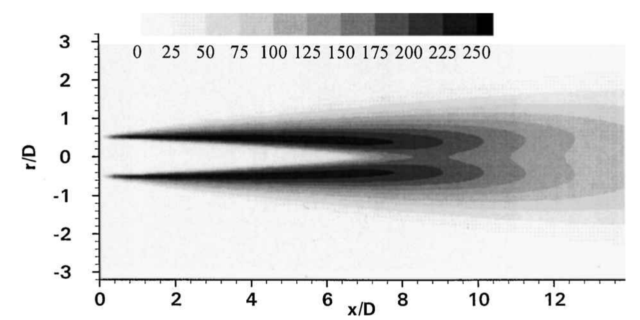

Fig. 4 Aerodynamic sources localization in a heated jet: $D = 50.8$ mm, $U_j = 136$ m/s, and $T_j = 786$ K.

Figure 3 shows that the jet mixing layers correspond to the maxima of turbulent kinetic energy. Production of turbulent energy is indeed related to velocity gradients that reach a maximum in the mixing layers and are negligible in the potential core and away from the exit nozzle. All of these indicate that suitable numerical predictions are obtained from the k–ε computation code used.

B. Sources of Sound Localization in a Heated Turbulent Jet

Localization models of acoustic sources issuing from the velocity fluctuations exist (e.g., Ref. 20), but acoustic source localization models from temperature fluctuations do not exist to our knowledge. Consequently, we use our estimations of source terms [Eqs. (7) and (17)] to carry out and compare the localization of both kinds of sound sources.

The localization of sound sources relative to velocity fluctuations in a hot jet is plotted in Fig. 4. We observe that they are located near the jet exit, along the potential core. They are spread over roughly 10 diameters in the axial direction and 1.5 diameters in the radial direction. In other words, they are located in the maxima regions of turbulent kinetic energy (see Fig. 3), which result directly from the modeling of this term source.

Figure 5 shows that the acoustic sources from entropic origin are located near the jet exit, along the potential core, like those of the aerodynamic sources. However, they are spread over a smaller distance in the axial direction (from 0 to 7 diameters) and over a slightly larger distance in the radial direction (for about 2 diameters). They

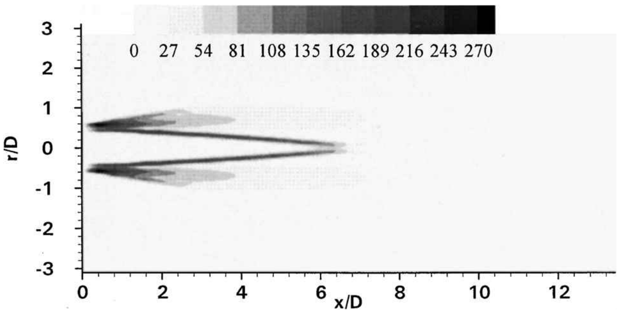

Fig. 5 Entropic sources localization in a heated jet: $D = 50.8$ mm, $U_j = 136$ m/s, and $T_j = 786$ K.

are also located in the mixing layers and the transition zone, near the jet exit. This is explained, among other things, by our entropic source model, based on the turbulent kinetic energy of which the maxima are located at these zones.

After this first global approach about the spatial distribution of the acoustic sources, which are located in the mixing and transition zones, like those in the unheated jets, the next section is devoted to the frequential features of the sound radiation.

C. Spectral Distribution of Acoustic Radiation of a Hot Jet

First, all parameters in the expression (23) of the coherence function to calculate the acoustic intensity spectrum of a subsonic heated jet have to be defined. Then the numerical estimation of the spectral distribution is achieved for different jets.

1. *Parameters of Spectral Distribution Simulation*

As already stated, the aerodynamic field is provided from a k–ε turbulence model, and the rms amplitudes of the acoustic source terms are directly deduced from this simulation. The acoustic intensity spectrum formulation shows the interactions between the sources in terms of a spatiofrequential intercorrelation function, in which two spatial scales and one frequential scale to be modeled appear. We choose to retain constant values of spatial scales to simplify the numerical estimation taking into account the anisotropy of the jet:

$$L_x = 3D, \qquad L_r = L_x/3 = D \tag{24}$$

The turbulence characteristic angular frequency is given by the following expression:

$$\omega_t = C_\omega \times 2\pi(\varepsilon/k) \tag{25}$$

where the dimensionless coefficient of proportionality C_ω is adjusted by comparison with experimental data.

The convection velocity is assumed constant and proportional to the jet axis velocity, as

$$U_c = 0.65U_j \tag{26}$$

2. *Numerical Model Adjustment*

An experimental configuration is reproduced numerically to compare and to adjust the numerical model. We carry out a simulation with respect to Tanna's experimental conditions,[9] i.e., the same diameter, velocity, and temperature of the exit jet. This configuration allows comparison with Tanna's experimental data to adjust the computed acoustic intensity. Such an adjustment is necessary because of the approximations made all along the theoretical development: far-field approximation and evaluation of physical quantities L_x, L_r, and ω_t from an order of magnitude estimation. Two constants of adjustment had to be defined: First C_ω, which acts on both the shape and the central frequency of the spectrum, and second, a dimensionless constant α, defined as $I_{\text{adjusted}} = \alpha \times I_{\text{computed}}$, acting on the amplitude of spectrum. The following constants values are obtained: $\alpha = 2.5 \times 10^{-2}$ and $C_\omega = 1.5$; then they are never modified. An imaginary and a real part appear in Eq. (22), and therefore, the modulus of the acoustic intensity is computed. Results are expressed in terms of acoustic intensity levels (decibel) with the

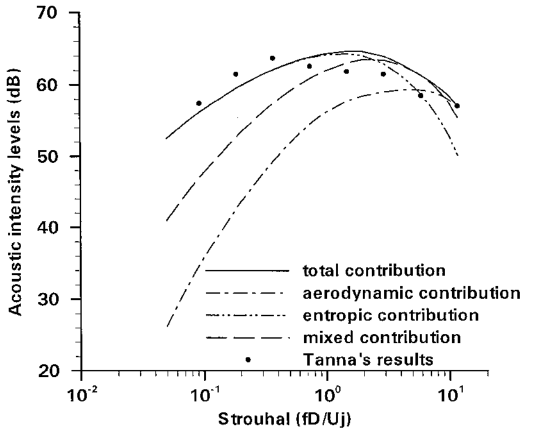

Fig. 6 Acoustic intensity spectra at 90 deg: $D = 50.8$ mm, $U_j = 136$ m/s, and $T_j = 786$ K.

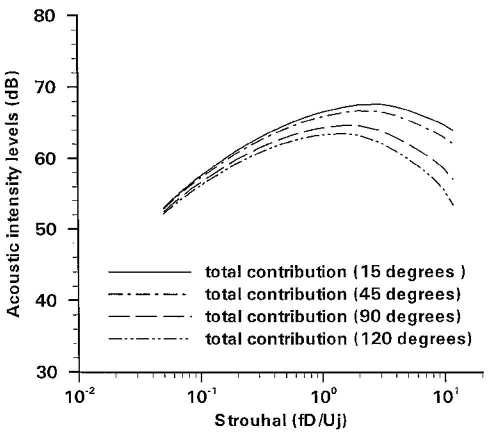

Fig. 7 Acoustic intensity spectra at different observer locations: $D = 50.8$ mm, $U_j = 136$ m/s, and $T_j = 786$ K.

reference $I_0 = 10^{-12}$ W/m^2, for an observer located at 72 diameters from the jet exit and at 90 deg from the jet axis (see Fig. 1).

Figure 6 provides intensity total levels computed with our model, Tanna's experimental results,[9] and in addition the separated contribution from every acoustic source to total radiated acoustic intensity levels. The obtained numerical spectrum is a broadband spectrum, with a peak frequency for a Strouhal number about 1.5, whereas the experimental results show a peak frequency for a Strouhal number about 0.35. This overestimation of the peak frequency results from the definition of the coherence function (23), as noticed by Bailly.[21] However, it appears that the shape of the spectrum is close to the measured one as the differences are less than 3 dB. We note that the aerodynamic contribution (linked to the velocity fluctuations) governs the spectrum in high frequencies and that the entropic contribution (linked to temperature fluctuations) dominates the spectrum in low frequencies. The entropic contribution is accountable for an important increase of sound levels in the low frequencies and, therefore, for the shift of the peak frequency to the low frequencies. The mixed contribution (linked to both fluctuation types) governs the spectrum in the middle frequencies. All of this agrees with experimental results, which show an increase in noise emission from the heated jet in comparison with the unheated one, and a shifting of peak frequency in low frequencies.

The acoustic radiation study is completed by computations from different observation locations at the same distance of the jet exit. Computations are carried out for 120-, 45-, and 15-deg observer angles, and results are shown in Fig. 7. It can be seen that the radiated acoustic intensity levels increase as the observer angle decreases. These features agree with those known about unheated jets. It must be emphasized that we do not observe a spectrum translation to low frequencies as the observer angle increases.

This is because the modeling does not take into account refraction effects, which occur at observer angles near jet axis and which

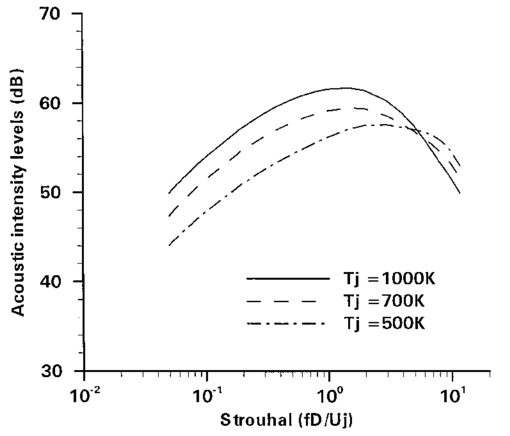

Fig. 8 Acoustic intensity spectra at 90 deg: $D = 25$ mm and $U_j = 125$ m/s.

become more important as frequency increases. Moreover, the convection factor in the Gaussian function in the acoustic intensity spectrum leads to a shift of spectrum to high frequencies, which is not realistic.

3. *Effects of Jet Temperature*

The preceding section was devoted to global features of sound emission from a heated jet: spectrum form, peak frequency, and evolution with observer angle. This section deals with temperature effects on the radiated spectrum. Figure 8 shows radiated spectra of three jets of different temperatures, for an observer located at 72 diameters and 90 deg from the jet axis.

Acoustic intensity spectral distribution varies in a consistent manner with temperature. Acoustic intensity levels in the low frequencies increase as the temperature jet increases. The jet with the highest temperature produces a sound emission 6 or 7 dB higher than that of the jet at the lowest temperature. Acoustic intensity levels in the high frequencies decrease as the temperature jet increases. This is explained by the decrease of jet local density, which is taken into account in the aerodynamic sources modeling. These two points clearly show that an increase in jet temperature leads to a shift of spectrum where the low frequencies are more pronounced. In addition to predicting correctly experimental trends observed by Tanna,[3] the orders of magnitude in variations of peak frequency and amplitude are obtained with quite a good accuracy. Indeed for similar temperature and velocity of turbulent jets, Tanna's measurements show a peak frequency shifting toward low frequencies in a ratio of $\frac{1}{3}$ and an increase of the amplitude of about 6 dB, whereas the simulations give a ratio of $\frac{1}{4}$ for the peak frequency, shifting with an increase of the amplitude on the order of 5 dB.

4. *Influence of Coherence Function Form on the Radiated Spectrum*

The preceding computations were carried out with the Gaussian form of coherence function used for unheated jets. As already stated, we know that it does not allow very good modeling of physical phenomena. This led Bailly[21] to suggest a hyperbolic form of the temporal correlation, and following this idea, we tried the spatiofrequential coherence function

$$\Gamma(y', \boldsymbol{\xi}, \omega) = \exp\left(\frac{-\pi\xi_x^2}{L_x^2}\right)\exp\left(\frac{-\pi\xi_r^2}{L_r^2}\right) \times \left[\pi \Big/ \beta\omega_t \cosh\left(\frac{\pi}{2\beta}\frac{C\omega}{\omega_t}\right)\right] \quad \text{with} \quad \beta = \frac{2}{5} \tag{27}$$

A computation with respect to experimental Tanna conditions[9] is carried out with the same spatial and frequential scales as defined earlier. Comparison with Tanna's experimental data is shown in Fig. 9, and a new adjustment factor value is determined ($\alpha = 10^{-1}$). It is shown that this function does not allow a better estimation of acoustic intensity spectrum radiated from a hot jet. The obtained spectrum strongly underestimates intensity levels in high

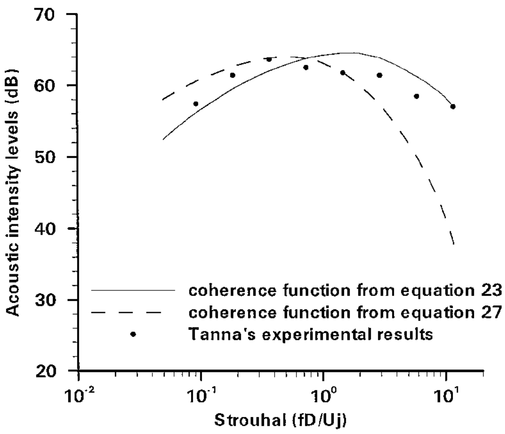

Fig. 9 Comparison of both coherence functions: $D = 50.8$ mm, $U_j = 136$ m/s, and $T_j = 786$ K.

frequencies and overestimates slightly those in low frequencies. However, the peak frequency occurs at a Strouhal number of 1, which represents a better estimation of experimental values.

The numerical model presented in the preceding section has allowed us to give some information about the spectral distribution of acoustic intensity radiated from a turbulent hot jet. The computational results show that this model provides a good prediction of the global features of the noise radiated by a hot jet: increase of sound levels in the low frequencies and shift of spectrum to the low frequencies. In addition, it could be used for studying the influence of jet temperature on sound emission.

IV. Conclusion

Some authors have pointed out the existence of an additional source of sound from jet temperature in a heated jet in comparison with an unheated one. This source appears to be linked with temperature fluctuations in a jet. The present work was devoted to modeling this acoustic source in terms of mean quantities given by a turbulent k–ε closure. Following the usual method of estimating acoustic radiation from an unheated jet, we worked out a calculation model of an acoustic intensity spectrum, in which interactions between acoustic source terms were represented by a spatiofrequential coherence function. The intensity spectrum shows three different contributions: The first is linked to the interactions between the velocity fluctuations, the second to the interactions of the temperature fluctuations, and the third to the interactions between the two preceding fluctuations. Results from our numerical model show that it is able to predict correctly global features of sound emission from hot jets. However, the limits of this model can be observed. On the one hand, the first-order closure used to estimate the velocity–temperature correlation allowed us only a coarse modeling of physical phenomena. On the other hand, it was necessary to adjust parameters, such as spatial and frequential scales, and the adjustment factors, as in all models based on a turbulent k–ε closure. To improve the acoustic radiation modeling of hot jets, another method to compute the aerodynamic fields can be envisaged, such as the large eddies simulation. In fact, this method is based on the simulation of large turbulent structures, which, in practice, dominate sound radiation of the flow. Therefore, it appears especially well adapted to an acoustic posttreatment.

References

[1]Zoppellari, E., and Juvé, D., "Reduction of Hot Supersonic Jet Noise by Water Injection," AIAA Paper 98-2204, June 1998.

[2]Lush, P. A., "Measurements of Subsonic Jet Noise and Comparison with Theory," *Journal of Fluid Mechanics*, Vol. 46, No. 3, 1971, pp. 477–500.

[3]Tanna, H. K., "An Experimental Study of Jet Noise—Part I: Turbulent Mixing Noise," *Journal of Sound and Vibration*, Vol. 50, No. 3, 1977, pp. 405–428.

[4]Lighthill, M. J., "On Sound Generated Aerodynamically—I. General Theory," *Proceedings of the Royal Society of London*, Vol. A211, No. 1107, 1952, pp. 564–587.

[5]Lilley, G. M., "The Generation and Radiation of Supersonic Jet Noise. Vol. IV: Theory of Turbulence Generated Jet Noise, Noise Radiation from Upstream Sources, and Combustion Noise. Part II: Generation of Sound in a Mixing Region," U.S. Air Force Aero Propulsion Lab., TR 72-53, Wright AFB, OH, 1972.

[6]Béchara, W., Lafon, P., Bailly, C., and Candel, S., "Application of a k–ε Model to the Prediction of Noise for Simple and Coaxial Free Jets," *Journal of the Acoustic Society of America*, Vol. 97, No. 6, 1995, pp. 3518–3531.

[7]Bailly, C., Candel, S., and Lafon, P., "Prediction of Supersonic Jet Noise from a Statistical Acoustic Model and a Compressible Turbulence Closure," *Journal of Sound and Vibration*, Vol. 194, No. 2, 1996, pp. 219–242.

[8]Fisher, M. J., Lush, P. A., and Harper Bourne, M., "Jet Noise," *Journal of Sound and Vibration*, Vol. 28, No. 3, 1973, pp. 563–585.

[9]Tanna, H. K., Dean, P. D., and Fisher, M. J., "The Influence of Temperature on Shock-Free Supersonic Jet Noise," *Journal of Sound and Vibration*, Vol. 39, No. 4, 1975, pp. 429–460.

[10]Lilley, G. M., "On the Noise from Jets," *Proceedings of the AGARD Conference on Noise Mechanisms*, CP-131, AGARD, 1973.

[11]Lilley, G. M., "The Radiated Noise from Isotropic Turbulence with Applications to the Theory of Jet Noise," *Journal of Sound and Vibration*, Vol. 190, No. 3, 1996, pp. 463–476.

[12]Goldstein, M. E., *Aeroacoustics*, McGraw–Hill International, New York, 1976, Chap. 2.

[13]Morfey, C. L., "Sound Radiation Due to Unsteady Dissipation in Turbulent Flows," *Journal of Sound and Vibration*, Vol. 48, No. 1, 1976, pp. 95–111.

[14]Berman, C. H., "The Generation and Propagation of Sound in Turbulent Flows," AIAA Paper 79-0573, 1979.

[15]Fulachier, L., and Dumas, R., "Spectral Analogy Between Temperature and Velocity Fluctuations in a Turbulent Boundary Layer," *Journal of Fluid Mechanics*, Vol. 77, No. 2, 1976, pp. 257–277.

[16]Abramovich, G., *The Theory of Turbulent Jets*, MIT Press, Cambridge, MA, 1963, Chap. 1.

[17]Ribner, H. S., "Quadrupole Correlations Governing the Pattern of Jet Noise," *Journal of Fluid Mechanics*, Vol. 38, No. 1, 1968, pp. 1–24.

[18]Lau, J. C., "Effects of Exit Mach Number and Temperature on Mean-Flow and Turbulence Characteristics in Round Jets," *Journal of Fluid Mechanics*, Vol. 105, No. 1, 1981, pp. 193–218.

[19]*Fluent User's Guide Version 4.3*, Fluent Inc., 1995.

[20]Davies, P. O. A. L., Fisher, M. J., and Barratt, M. J., "The Characteristics of the Turbulence in the Mixing Region of a Round Jet," *Journal of Fluid Mechanics*, Vol. 15, No. 3, 1963, pp. 337–367.

[21]Bailly, C., "A Statistical Description of Supersonic Jet Mixing Noise," AIAA Paper 97-1575, May 1997.

P. J. Morris
Associate Editor